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Occupational choice and the private equity premium puzzle

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Abstract

This paper suggests a resolution to what has come to be known as the 'private equity premium puzzle' (Am. Econom. Rev. 92(4) (2002) 745–778). We interpret occupational choice as a dynamic portfolio choice problem of a life-cycle investor facing a borrowing constraint, stock market participation costs and imperfect information about the profitability of potential businesses. Information is imperfect, because only entrepreneurs observe their own business risk realizations and there is a fixed cost of starting a business. Using numerical techniques we find that the model generates the empirically observed return structure for private and public equity with standard CRRA-preferences and fully rational expectations.

JEL classification: G11; D91

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1. Introduction

Recently, Moskowitz and Vissing-Jorgensen (2002) (referred to as MVJ from now on) have documented the risk-return structure and the portfolio allocation of private

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equity in the US. They find that average private equity ex post returns are roughly equal to average ex post returns on public equity and that the two series are highly correlated with each other. At the same time, investment in private equity is extremely concentrated with most entrepreneurial households investing more than half of their private equity holdings in a single firm and returns at the individual firm level are highly variable and skewed to the left. In addition, the majority of the equity capital of a single firm is regularly held by a single private investor who also has a management interest in the firm. The authors conclude that, given the concentration in individual portfolios and the large idiosyncratic risk associated with private equity, investment in private firms seems to be dominated by investment in publicly traded equity for risk-averse individuals and term this finding a 'private equity premium puzzle'.

Examining the empirical results in more detail, there are two different dimensions of this puzzle. The 'corporate finance'-dimension is why most private firms are controlled by a single shareholder. As shown by Bitler et al. (2005), a simple moral hazard model goes a long way toward explaining this finding. The 'portfolio choice'-dimension seems to be more difficult to explain. The issue is, why entrepreneurs are willing to hold the equity of a single firm given the unfavorable variance-return characteristics of the asset. As observed by MVJ this behavior violates the participation constraint in the moral hazard game.

In the following, we propose a resolution to the 'portfolio-dimension' of the puzzle by showing that the observed structure of ex post private equity returns and the concentrated portfolio allocations of private equity emerge from a simple model of occupational choice over the life-cycle. We find that the realized average rate of return on non-traded private equity is slightly below the realized average return on public equity of 7.25%. The value-weighted index return of realized private equity returns is 6.6%. At the same time, cross-sectional returns to private equity are very volatile and for agents holding private equity, these holdings represent a large share of their total wealth. In addition to these findings, our model also replicates other facts about entrepreneurship and private businesses reported by MVJ and others.

Just as MVJ did, we take the decision to become an entrepreneur as being equivalent to the decision of investing in private equity. When giving this interpretation to the choice of becoming an entrepreneur however, it is also important to consider the special kind of investment environment in which this choice is embedded as was pointed out also by Polkovnichenko (2003). We argue that the appropriate setting for studying entrepreneurial choice is a dynamic life-cycle model with finitely lived agents who in each period can decide to be entrepreneurs or workers. Polkovnichenko (2003) considers a static model instead. Households cannot borrow against future labor income and cannot sell short any asset. Our model creates a substantial premium of public equity over the riskless asset due to the existence of participation costs in financial markets as in Vissing-Jorgensen (2002) or Campbell et al. (1999). Becoming an entrepreneur in our model is equivalent to paying a fixed startup cost and investing a positive amount into an asset with a highly persistent, idiosyncratic rate of return. Hence, our entrepreneurs take on a substantial amount of idiosyncratic income risk. Being an entrepreneur does carry a substantial benefit however, which is the additional information on the own business's return available to the entrepreneurs. While all agents are informed about the unconditional distribution of the returns to entrepreneurship, only entrepreneurs observe their own business risk realization and can form a conditional expectation about future returns. This additional information allows entrepreneurial households to make better asset allocation decisions.

Because this information is more valuable at the beginning of the life-cycle, our model predicts that average rates of return on private equity depend on the age of the entrepreneur and the age of the business.[1] Clearly, the existence of startup costs which will be amortized over time implies that young businesses have lower average rates of return than mature businesses. In addition, very young businesses have a larger probability of drawing a bad business risk realization, because upon startup this information is not known to the entrepreneur and business risk is highly serially correlated. Third, young businesses tend to be held by young agents. These are the main reasons why young businesses have lower average rates of return than the older, more established businesses.

Young agents require lower average rates of return to start a business than older agents. The persistence in private equity returns and the additional information about their idiosyncratic return available to entrepreneurs make it attractive for young agents to start a business even if the one-period expected return is low. Young entrepreneurs implicitly hold an exit option that is more valuable than the exit option of older entrepreneurs because time to expiration is longer. This information effect is in addition to the 'human capital effect' demonstrated by Polkovnichenko (2003) which is also present in our model. As a result of both, young agents prefer investing in private equity rather than public equity, even if average expected rates of return are substantially lower. The crucial factors for our results are therefore the startup costs associated with private businesses and the persistence in private business returns, the human capital endowment of households and the participation costs in the stock market.

The next section provides a brief overview of the literature on occupational choice and optimal portfolios in life-cycle models. Section 3 presents the model in detail. In Section 4 we discuss calibration issues and present our baseline results. Section 5 discusses the determinants of entrepreneurship in our model and reports some sensitivity analysis. Section 6 concludes and points to interesting research topics in the future. The appendix contains a description of our computational procedure.

2. Related literature

The theoretical literature on entrepreneurship has been booming recently after receiving surprisingly little attention for a long time. Following Schumpeter's treatise on economic development and entrepreneurship (Schumpeter, 1934), few academic

[1] There is some empirical support for this testable implication in the SCF data but estimated average returns by age are hard to compare because the variance and the number of observations declines with age.

papers have studied this phenomenon. Laffont and Kihlstrom (1979) consider risk aversion as the main determinant of becoming an entrepreneur in a static general equilibrium setting, but fail to provide convincing empirical evidence for their hypothesis. In an interesting and challenging paper, Banerjee and Newman (1993) analyze the effect of liquidity constraints on occupational choice and growth dynamics. They show that various paths of economic development are possible for economies with the same technological characteristics and that ultimate outcomes crucially depend on the initial distribution of wealth. Cagetti and DeNardi (2002) study the decision of becoming an entrepreneur in an overlapping generations setting. They focus on the wealth distribution rather than the returns to entrepreneurship and use a more stylized, general equilibrium model without occupational choice where entrepreneurs operate in a secluded sector, allowing them to jointly determine the rates of return and the amount of private equity capital accumulated. Our model instead takes the distribution of rates of return on entrepreneurship as exogenous, but endogenously determines the share of wealth invested in private equity and the occupational choice at each point in time. Focusing on the public equity premium, Heaton and Lucas (2000) study portfolio choice of households in the presence of background risk introduced through private equity investment.

Other than MVJ, there also exists little empirical work on the return distribution of private equity. Empirical studies of entrepreneurship have focused instead on finding determinants of the decision to become an entrepreneur or be self-employed. Evans and Jovanovic (1989) were among the first to study the determinants of entrepreneurship, focusing in particular on the effect of receiving a large gift or bequest on the probability of becoming an entrepreneur. Subsequent studies such as Holtz-Eakin et al. (1994), Hurst and Lusardi (2004) and Hamilton (2000) enlarge the set of determinants considered, but find similar results concerning the positive effects of large positive income shocks.

Two papers that are closely related to ours are those of Polkovnichenko (2003) and Willen (2003). Polkovnichenko (2003) shows in a static model with mean-variance preferences, that young investors require only a small expected return premium on private equity over public equity because of their large human capital holdings. Essentially, investing even a large fraction of their tangible wealth into a single business does not constitute a large consumption risk for these households, because their future wages are not at risk, even if the business generates low returns. While making an important point, the main problem with this paper is that it does not address the issue of why holding public equity does carry a premium, while private equity apparently does not. Willen (2003) provides an alternative explanation for the private equity premium puzzle based on the relaxation of credit constraints due to collateralizable private business assets. His framework requires that 'private equity capital is much more collateralizable than public equity investments'.

A large literature exists on optimal saving decisions over the life-cycle, if liquidity constraints are binding and one asset is available. Auerbach and Kotlikoff (1987) and Hubbard and Judd (1987) were among the first to study this issue. More recent studies including a portfolio-choice component, participation costs and stochastic labor income are by Campbell et al., 1999, Laibson et al., 1998, and Davis et al. (2002). Campbell et al. (1999) study the implications of investing retirement assets in the stock market rather than government bonds. Laibson et al. (1998) build a detailed model of the US economy taking into account heterogeneity in wealth and education levels, but focus on the optimal saving behavior of consumers with time-inconsistent preferences and its implications for public pension schemes. Davis et al. (2002) build a model with differential borrowing rates for various asset classes and study optimal lifetime asset allocation in this context.

Finally, there is a huge literature on asset pricing anomalies and the related portfolio allocation puzzles. Participation costs in the stock market have recently been shown to provide a plausible explanation for the historically high Sharpe ratio of public equity returns by Vissing-Jorgensen (2002) and Wang (2003). Constantinides et al. (2002) discuss ways to account for the observed premium on public equity in rational expectations models and favor limited stock market participation. While there is no consensus yet about whether stock market participation costs are the correct explanation to this puzzle (see Polkovnichenko (2004) for a different opinion), incorporating them into life-cycle portfolio choice models does improve the characteristics of these models substantially and we have chosen to follow the same route as well.

3. Modeling occupational choice

Like Campbell et al. (1999) and Laibson et al. (1998) our life-cycle problem is set in a partial equilibrium context, taking the stochastic processes of asset returns and wages as given. Individuals have finite, stochastic lifetimes and we set a fixed retirement date. There is no bequest motive. There are three types of assets available in the economy: a frictionlessly traded riskless asset with a fixed rate of return denoted by B_{it}, a publicly traded risky asset subject to aggregate return risk denoted by S_{it} and non-traded private equity capital invested in a single firm which is managed by the same entrepreneurial household carrying both aggregate and idiosyncratic risk and denoted by Q_{it}.

Household $i \in \{1, 2, \ldots, I\}$ maximizes the discounted sum of utility derived from consumption $u(c_{it})$ net of participation costs over a finite horizon of T periods:

$$\max_{c_{it}, B_{i,t+1}, S_{i,t+1}, Q_{i,t+1}} E_0 \left[\sum_{t=1}^{T} \delta^t \left(\prod_{j=1}^{t} p_j \right) (u(c_{it}) - \mathbf{1}(S_{it} > 0)\Psi) \right], \tag{1}$$

where δ is a discount factor equal for all households and p_t denotes the probability of being alive at date t, conditional on being alive at date $t-1$. The fixed utility cost of investing in the stock market is denoted by Ψ. This cost is incurred in any period in which the household invests in the stock market and in our interpretation represents opportunity costs of time and effort spent in obtaining and processing market information. Wang (2003) and Bilias and Haliassos (2004) have recently shown that participation costs in the stock market improve the portfolio choice implications of life-cycle asset allocation models. Utility maximization is subject to the following budget constraint.

$$c_{it} = (1-\theta)w_{it} + (1+r)B_{it} - B_{i,t+1} + (1+d_t)S_{it} - S_{i,t+1} + (1+v_{it})Q_{it} - Q_{i,t+1} - \mathbf{1}(Q_{it}=0, Q_{i,t+1}>0)\Phi \tag{2}$$

and short-sale constraints on all available assets

$$B_{it} \geqslant 0, \quad S_{it} \geqslant 0, \quad Q_{it} \geqslant 0 \quad \forall t. \tag{3}$$

Exogenous labor income w_t has an age-dependent deterministic component

$$\bar{w}_t = \begin{cases} g(t) & \text{for } 1 \leqslant t \leqslant P, \\ \bar{b} & \text{for } P < t \leqslant T, \end{cases} \tag{4}$$

where P denotes the fixed retirement age. In addition, there is uninsurable idiosyncratic income risk ε_{it} such that

$$\log w_{it} = \log \bar{w}_t + \omega_t + \varepsilon_{it}. \tag{5}$$

The stochastic income component is described by

$$\varepsilon_{it} = \phi\varepsilon_{i,t-1} + \eta_{it}, \tag{6}$$

where η_{it} is an idiosyncratic i.i.d. shock distributed as

$$\eta_{it} \sim N\left(0, \sigma_\eta^2\right) \tag{7}$$

and ω_t is the aggregate i.i.d. shock distributed as:

$$\omega_t \sim N\left(0, \sigma_\omega^2\right). \tag{8}$$

The riskless financial asset, B_{it}, yields a certain net rate of return r. The net rate of return on public equity, S_{it}, is denoted by d_t and equal for all households. We assume that it is uncorrelated over time

$$1 + d_t = \exp\left(\bar{d} + \omega_t\right). \tag{9}$$

The asset corresponding to investment in the own private firm, Q_{it}, yields an idiosyncratic net excess return over public equity

$$v_{it} - d_t = \exp\left(\bar{v} + \rho_\varepsilon\varepsilon_{it} + \zeta_{it}\right) \tag{10}$$

which is the exponential of a sum of three components: an average guaranteed return, $\bar{v}$, a skill component perfectly correlated with exogenous income risk, $\rho_\varepsilon\varepsilon_{it}$, and an idiosyncratic business risk component, ζ_{it}. This formulation ensures that public equity returns are highly correlated with aggregate private equity returns as in MVJ. The idiosyncratic skill component and the aggregate risk component also affect the exogenous income process which implies that there is a sizeable positive correlation between wage income and private equity returns. The business risk component is orthogonal to the skills component and evolves according to

$$\zeta_{it} = \psi\zeta_{i,t-1} + \xi_{it}, \tag{11}$$

where ξ_{it} is another i.i.d. shock distributed as

$$\xi_{it} \sim N\left(0, \sigma_{\xi}^2\right) \tag{12}$$

and every newly created business receives an initial draw from the steady-state distribution of ζ. Investment in private equity is subject to startup costs of becoming an entrepreneur denoted by Φ.

A crucial feature of the model are the assumptions relating to the information on the uncertainty realizations available to the agents. We assume that each household can observe the realization of idiosyncratic wage income risk at each point in time and that business risk is unobserved prior to starting a business. This implies that only entrepreneurs, defined as having a portfolio with a strictly positive level of private equity, can use conditional distributions of returns in their optimal decisions by applying transition probabilities over the states of business risk. Non-entrepreneurs are facing the unconditional distribution of business risk.

The structure of this problem is quite close to a standard dynamic portfolio choice problem for a finite horizon investor introduced by Samuelson (1969) and Merton (1969). The distinguishing features of our model are the existence of startup costs, stock market participation costs, and the informational assumption that returns to the entrepreneurial asset are observed only when its share in the portfolio is strictly positive. These assumptions together with the short-sale constraints require the use of numerical techniques to find the solution of the problem.

4. Calibration and baseline results

We set the maximum lifetime of agents in our model to 89 years and assume that agents enter the model at age 20. In order to economize on computational resources, we choose a period length of 3 years and hence arrive at a total lifetime of 23 periods. The mandatory retirement date is the end of period 14, which is equivalent to an age of 62 years. Death probabilities are taken from the lifetables of the US National Center for Health Statistics, which report conditional survival probabilities[2].

Agents are assumed to maximize a standard instantaneous utility function of the CRRA-class,

$$u(c_{it}) = \frac{c_{it}^{1-\sigma} - 1}{1 - \sigma} \tag{13}$$

setting the coefficient of relative risk aversion σ equal to 2 and the discount rate δ to 0.97. These values are common in the literature (see Conesa and Krueger, 1999; Campbell et al., 1999; Cagetti and DeNardi, 2002) and consistent with estimates by Gourinchas and Parker (1999) in their analysis of lifetime consumption profiles. The riskless interest rate r is set to 2.5% annually and the expected public equity return is set to 7.25% ($\bar{d} = 0.07$) with an annual standard deviation of returns of 10%. These

[2]US National for Health Statistics, 1994.

Table 1
Fixed parameters

Parameter	Value
σ	2
δ	0.97
r	0.025
$\bar{d}$	0.07
σ^2_ω	0.01
θ	0.125
ϕ	0.688
σ^2_η	0.052

values imply a Sharpe ratio for stock market excess returns of 0.475 which is in line with the historical average (Table 1).

We take the calibration of the wage income process from the detailed study by Laibson et al. (1998).[3] These authors estimate age–income profiles for three educational groups from PSID data by regressing log income on powers of age and some control variables. We select the median educational group as being most representative for the average household

$$g(t) = \exp(8.835 + 0.058t - 0.017t^2/100 - 0.055t^3/10000). \quad (14)$$

We convert their results obtained for annual data to a 3-year frequency by time aggregation. The autocorrelation coefficient ϕ of the stochastic wage process is equal to 0.688 for annual data, which implies a coefficient of 0.326 for our calibration. The variance of the innovation to log income σ^2_η is 0.052 for annual data, resulting in a value of 0.0883 for our modeling frequency. We approximate the social security system by a proportional tax on exogenous income, choosing a tax rate θ of 12.5% and compute retirement benefits from assuming a balanced social security budget. Draws of initial wealth are taken from a lognormal distribution, using the empirical mean of the wealth distribution for the youngest cohort (equal to \$23,183) and the corresponding coefficient of variation of 6.53 given in Budria et al. (2002). In the aggregation we used population weights from the 1998 issue of the Current Population Survey (CPS), truncated below age 20, assuming a long-run real income growth rate of 1%.

We now discuss the calibration of private equity returns and startup and participation costs. Unfortunately, little direct empirical evidence is available for private equity returns. Private companies are not obliged to disclose this kind of information and even the evidence that is available from tax filings or surveys is not very reliable. As a consequence, it is difficult to come up with reliable estimates for

[3]These authors estimate a relatively easy way to implement household life-cycle income process which is useful also for the purpose of our analysis. In their specification, income is uncertain also in retirement, but varies much less in absolute terms than in the working period. Their median educational group are the high school graduates.

the parameters of the private equity return process. We therefore resort to calibration and choose the deterministic component of the private equity return $\bar{v}$, to obtain an empirically plausible share of entrepreneurs in the population. Gentry and Hubbard (2000) consider various definitions of entrepreneurship and report shares of 8.7–11.5% in the population with 8.7% being their preferred interpretation. In our baseline simulations, the share of entrepreneurs in the population is 12.5%. The total return on private equity also includes an aggregate risk, a skill and business risk component. We choose the autocorrelation coefficient ψ, the weight of skills in the return distribution and the variance of the innovation to business risk σ_{ξ}^2 in order to achieve a relatively wide distribution of possible asset returns. Our baseline calibration sets the annualized values for ψ equal to 0.867, for σ_{ξ}^2 equal to 0.04, and ρ_{ε} equal to 0.4 (Table 2). These values translate into a span from −56% to 155% for the unconditional distribution of private equity returns in the calculation using a grid that is 4 standard deviations wide.

Transaction costs are chosen to be of a plausible magnitude compared to wages. We set the startup cost parameter Φ equal to 1.54, equivalent to about one annual income of a young household. High startup costs are the main deterrents for entrepreneurship in association with the borrowing and short-selling constraints. Without startup costs, households frequently invest limited amounts of wealth in private equity to remain informed about business risk realizations and increase these holdings if returns are high. Consistent with evidence by Evans (1987) these imply that early-in-life entry rates into entrepreneurship are increasing with age. We choose per period stock market participation costs that are constant in utility terms and interpret these costs as opportunity costs of gathering and processing information about publicly traded companies. The monetary equivalent of these costs for a household with an annual consumption of 20,000US$ is 3,530US$. The following section presents our baseline results and provides an analysis of the model based on the computed policy functions.

4.1. Baseline results

Given these baseline parameters our model produces private equity holdings roughly equal to holdings of the riskless asset while public equity holdings are about $\frac{2}{3}$ of total asset holding. The average age of private businesses is 11.0 years, and the

Table 2
Calibrated parameters

Parameter	Value
$\bar{v}$	−0.06
ψ	0.867
σ_{ξ}^2	0.04
ρ_{ε}	0.4
Φ	1.54
Ψ	0.025

average age of entrepreneurs is 47.0 years. Wealth distributions are generally skewed to the left for all age groups and both entrepreneurs and non-entrepreneurs. The survival rates are low for young firms and younger households invest into private equity while older households prefer public equity investments.

We calculate private equity returns by analyzing the entrepreneurs population in a random sample of households which is constructed by using the empirically observed weights of the US population in 1998 and compute them as they are defined by MVJ on p. 769:

$$ret_{it} = v_{it} + \left(\frac{\text{value of business at time } t}{\text{value of original investment}}\right)^{1/(\text{years since founded})} - 1, \qquad (15)$$

where 'Value of business at time t' corresponds to the households' wealth invested in the business and 'Value of original investment' includes startup costs and the initial investment. As MVJ observe, this measure does not take into account subsequent investments or disinvestments but does provide a useful approximation if there is not a strong trend in subsequent investment rates.

The cross-sectional average return equals 1.5% annually (4.43% for a 3-year horizon) and hence is significantly below the average return on public equity which is 7.25%. The return on an index of all private equity held which gives less weight to smaller and less profitable businesses is also somewhat lower than the average return on public equity and calculated as 6.6% annually (21.14% for a 3-year horizon). At the same time, the cross-sectional ex post-return distribution is very wide, ranging from −142% to 162% for a 3-year horizon, with a standard deviation of 0.36. Hence, although the average entrepreneur does NOT earn a rate of return above the public equity, and holding private equity is associated with substantial idiosyncratic risk, agents are willing to take the risk of entrepreneurship (Table 3).

A central aspect of the model is the fact that average rates of return on private equity vary systematically with the age of entrepreneurs and the age of the firm. Young agents with high skills require low average rates of return to start a business, since by starting a business they can use their skills more productively and improve future asset allocation by gaining information about returns to their private business. Later in life, agents are wealthier on average, but they require larger rates of return to start a business because they are able to exploit high rates of return from a shorter period of time and as retirement nears, they become effectively more risk-averse (Fig. 1).

Table 3
Baseline results

Cross-section of private equity returns			Index return on private equity	Mean return on public equity	Average portfolio share of private equity
Mean return	Median return	Std. dev.			
1.5%	0.3%	0.36	6.6%	7.25%	77.3%

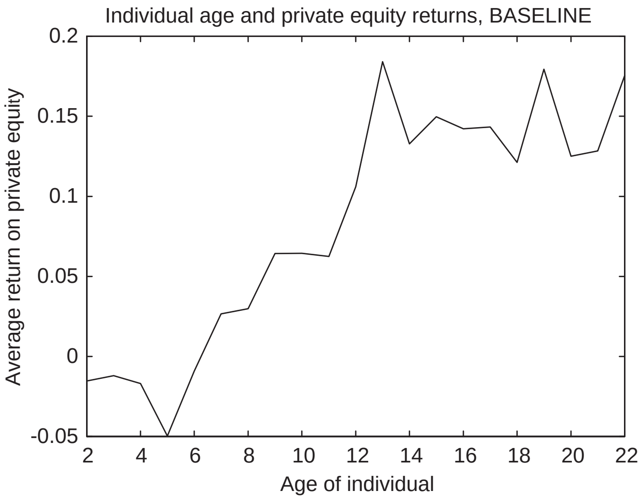

Fig. 1. Private equity returns by household age.

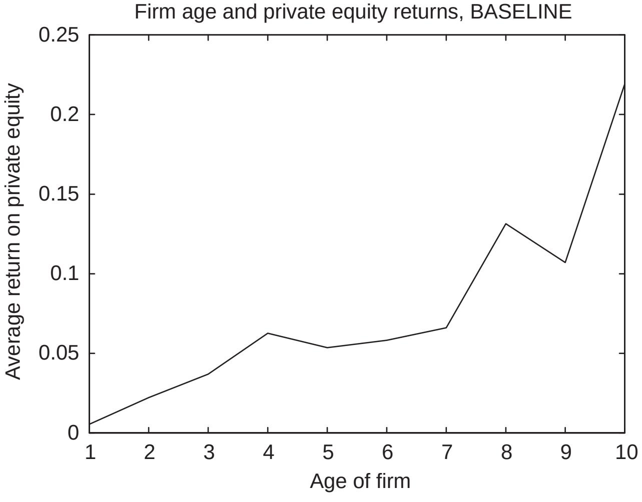

Fig. 2. Private equity returns by firm age.

The model also implies that young firms have relatively low rates of return for various reasons. First, the startup costs which are gradually amortized over time imply that young businesses have lower average rates of return than mature businesses. Second, very young businesses have a larger probability of drawing a bad business risk realization, because upon startup this information is not known to the entrepreneur. Third, young businesses tend to be held by young agents. Average firm age in our baseline calibration is 11.0 years and only slightly higher than the 10.7 years reported by MVJ (Fig. 2).

The following section is dedicated to an analysis of the determinants of becoming an entrepreneur and an exploration of the mechanism generating the results presented above.

5. Interpreting the results

We have seen that young agents require low average rates of return to start a business. The next section characterizes the young households which are willing to take entrepreneurial risk as those which are wealthier and more skilled. These households have a double advantage from taking entrepreneurial risk: they can use their human capital more productively since private business returns also depend on the skill realizations, and they obtain additional information about future business returns allowing them to better allocate their financial wealth in the future. Later in life, business startups are much less frequent because households are wealthier on average and the fixed participation costs reduce the return on public equity by less. Also, the gains from entrepreneurship are smaller the shorter the horizon of the household. Hence, these agents require larger average rates of return on private equity to remain invested in private equities or start a business. During retirement, investment in riskless assets becomes more attractive compared to both public and private equity because a certain component of lifetime wage income is significantly smaller and the slope of the earnings profile decreases which lowers agent's demand for risky assets (see Campbell et al., 2003). In the next three subsections we analyze the startup decision of households, the portfolio allocations over the life-cycle and provide some insightful experiments based on removing persistence in business risk and startup costs. We begin by analyzing the startup decisions of households.

5.1. Entry and exit rates

Our model generates startups that are relatively concentrated early in life from age 20 to about age 50. The entry rates are slightly increasing in this part of the life-cycle due to the startup costs and the liquidity constraint. This is consistent with empirical findings by Evans and Jovanovic (1989) who investigate startup decisions in a sample of white males. The characteristics of households starting a business implied by the model are also quite appealing. Households starting up a business are both more skilled and wealthier than the population average. This corresponds to results in the empirical literature (Holtz-Eakin et al., 1994; Hurst and Lusardi, 2004; Hamilton, 2000) which has established that large positive income shocks have a positive effect on the probability of becoming an entrepreneur in the future and that wage income is positively correlated with the propensity to start a business. Our model generates both of these results.

Fig. 3 displays the startup decisions for non-entrepreneurial households for various ages, wealth levels and skill realizations. For a given wealth level and skill

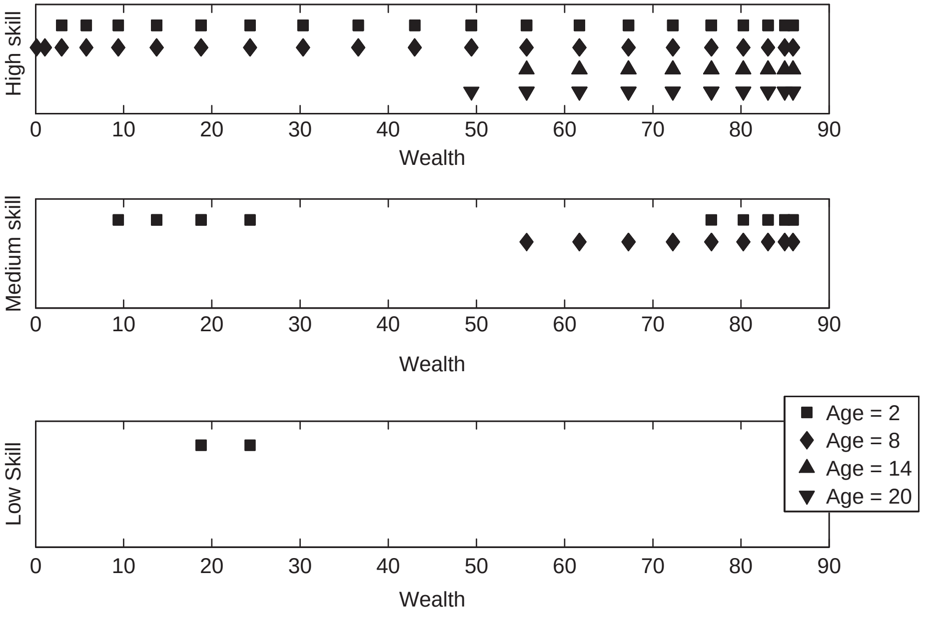

Fig. 3. Startup decision by age.

realization agents are generally more likely to start a business, if they are younger. At ages 2 and 8, households with the median skill realization start a business if their wealth is high enough. At age 2, agents also start a private business for some intermediate wealth levels even if their skill realization is low and high skill realizations lead to startups for most wealth levels in young age and for higher wealth levels in old age. Conditional on age, startup decisions and hence initial investments in private equity are positively correlated with current income and current wealth of households.

Exiting entrepreneurship not surprisingly is not that tightly related to skills or wealth. Rather, the life-cycle aspect of investment plays a prominent role there. Entrepreneurs in our model do not face any transaction costs of selling their business and we therefore see entrepreneurship rates drop relatively quickly prior to and at retirement. Households in this age group prefer to hold most of their wealth in public equity. Only after retirement, more and more agents invest a larger share of their wealth into the safe asset. This feature is a consequence of the increase in 'effective risk aversion' that life-cycle investors with risky income streams face as they are aging and the certain component of their wage income shrinks. This effect is well documented in Campbell et al. (2003).

5.2. *Age profile of public equity versus private equity investors*

A rather interesting result is also provided by the age profile of entrepreneurship and public equity investors. The empirical literature has established that young households tend to invest less in stocks than older households for any cohort (see Guiso et al., 2001). The majority of public equity holdings is concentrated in the age group of households with a head of 55 years and older. On the other hand, MVJ report that 75% of entrepreneurs are less than 55 years old which implies that younger households invest in private equity more frequently than older households do. Because older households investing in private equity tend to be richer on average, this does not imply that the majority of private equity is held by younger households, however.

Interestingly, the combination of startup costs, persistence in business returns and participation costs in the stock markets in our model yields qualitatively similar results. In the baseline case, we find that young households tend to invest in the riskless asset or private equity. Only 9.3% of households below age 56 hold public equity in our model. Consistent with the empirical data, our model implies a hump-shaped stock market participation ratio. From age 50 to 80 stock market participation rates are highest. Toward the end of the life-cycle stock market participation rates decline again. Holding age constant, stock market participation is increasing in wealth in increasing for all ages.

Somewhat different results are obtained for entrepreneurial choice. The share of entrepreneurs in each age group is monotonically increasing up to age 47 and then stabilizes and declines as agents approach retirement age and public equity investment becomes more widespread. While the fixed startup cost and the short-selling and borrowing constraints seem to capture relatively well the financing environment that entrepreneurial households face, our model probably allows entrepreneurs to divest from their business too easily. We therefore observe a relatively steep decline of the entrepreneur share around the retirement age and an average share of entrepreneurs in retirement of 5%.

5.3. Eliminating persistence in business risk

Both, the persistence and volatility of private equity returns play an important role in the model. Unfortunately, there is no information about the persistence of business returns to be gained from the SCF since it is not a panel study. Quadrini (1999) studies the dynamics of the wealth distribution and the savings choices of entrepreneurs and suggests that the persistence in total returns to private equity is relatively high. In order to check the sensitivity of our results to the chosen calibration and to gauge the importance of persistence in business risk for our results, we have computed a run without persistence in business risk ($\psi = 0$). This implies that the only source of serial correlation in private equity returns is the skill component. One important implication of the absence of serial correlation in business risk realizations is that the conditional and unconditional distributions of business risk are equivalent. Hence, there is no informational advantage for entrepreneurs concerning future business returns.

The results of this experiment are quite intriguing. It turns out that without the informational advantage of entrepreneurship, only very few households (less than 0.5%) ever become entrepreneurs and invest in private equity. The decline in private equity investment is especially pronounced for the young and middle-aged households. Households up to age 30 do not invest in private equity at all and entry rates for households up to age 65 are below 0.1%. These households prefer to accumulate wealth in the low-yielding riskless asset rather than paying the fixed startup cost and investing in risky private equity. Interestingly, this is true even for high-skill households which face a next period expected rate of return on private equity investment of 4.5% (annualized rate). Toward the end of the life cycle the persistence of business risk is less important and the decline in entry rates with respect to the baseline is less dramatic for these ages. Private equity holdings are nevertheless marginal at less than 0.5% of total wealth. Without private equity investment, most young households prefer investing in the riskless asset and only the wealthiest ones also invest in public equity. Again, stock market participation increases with age.

5.4. Eliminating startup and participation costs

We have argued above that startup costs are the main deterrent to investment in private equity. If there are no startup costs, households can costlessly switch in and out of private equity. There is still an informational advantage of entrepreneurship, but the direct costs of finding out about one's business risk realization are zero. Hence, households frequently invest a limited amount of their wealth in a private business to find out about the business's return. If the return is high they invest more, if the return is low they exit and try a new business next period. Eliminating startup costs therefore leads to a large amount of 'experimentation' in the economy. There is a large share of 'short-run entrepreneurs' in the population (the average age of businesses is only 1.7 periods which implies that few businesses survive for more than one period) and most households start more than one business in their life. Consequently, in the cross-section there is a very high entrepreneur share (about 50%) and entry and exit rates into and from entrepreneurship are very high for the young and middle aged (with a magnitude of about 30% on average) and decline gradually with age.

Further, also eliminating the participation costs and the persistence in business risk, we are left with a standard portfolio choice model with background risk. The results for this model are straightforward and follow from the fact that with standard risk aversion and a realistically calibrated equity premium, households invest almost all their wealth in publicly traded equity. The consumption profiles are much more homogeneous in the case of little or no private equity investment. This is a simple consequence of the fact that returns to public equity are i.i.d. in our model and that the stationary distribution of public equity returns has a much lower variance than the stationary distribution of unconditional private equity returns.

6. Conclusions

We have shown that the characteristics of private equity returns and portfolio allocations documented by Moskowitz and Vissing-Jorgensen (2002) emerge from a simple model of occupational choice over the life cycle. More specifically, the average return on a single business is significantly below the average return on a public equity index and cross-sectional returns on private businesses are much more variable. Nevertheless, agents holding private equity allocate a large share of their total wealth to a single business. Taking into account the size of private equity investments, we find that the value-weighted index return on private equity holdings is only slightly below the average return on a public equity index. A crucial characteristic of our model is that average private equity returns are increasing in both, the age of the entrepreneur and the age of the business. The key assumptions for this result are the existence of startup costs for privately held businesses, high persistence of private business returns and imperfect information about the profitability of potential businesses. The positive correlation between private equity returns and wage income makes private equity investment attractive for high-skill households. Additionally, households investing in private equity in our model have an informational advantage because they can observe their business risk realization. If business risk realizations are persistent this informational advantage improves future asset allocation and increases the value of entrepreneurship especially for the young households.

We find that the main determinants for becoming an entrepreneur are the agent's age, wealth and skills. Young agents with high skills or high wealth hold private equity, despite the low average return realization. Older agents are less likely to start up a business, they do so only when the expected rate of return is quite high. Our model generates an equity premium through the assumption of fixed stock market participation costs. One implication of this assumption in our model is that young households tend to invest in the riskless asset or private equity, while public equity holdings are concentrated in the age group of households age 55 and older. The amount of private equity investment by young households is highly sensitive to the persistence of private business returns because this affects the value of their informational advantage. If startup costs are removed, there is a lot of experimentation in the economy as households try to find out about their business risk realization by investing small amounts into a private business. Even though the focus of this paper is entirely on explaining the observed return structure on private equity the model is able to address other interesting topics such as the effect of pension system design on entrepreneurial activity, or the decision of entrepreneurs to take their private business public through IPOs. We leave these questions as potential topics for further research.

Appendix. Numerical procedure

The numerical method we use is finite state, finite horizon dynamic programming. We discretize the state space, defined over total wealth, portfolio composition and uncertainty states. Fineness of the grid basically determines computation time, which is linear in lifetime and quadratic for each asset in the number of gridpoints considered. We use a continuous approximation of the value function in the wealth dimension based on Chebyshev polynomials to improve the precision of our results using 20 gridpoints. The portfolio shares are approximated by an asymmetric grid with a total of 15 gridpoints. Each of the uncertainty states is approximated by a 3-point grid.

Given some discretization of the state space, for each point in time $t = 1, 2, \dots, T$, optimal policy rules are computed, describing the optimal consumption in t and the optimal portfolio allocation in $t + 1$. These rules are computed recursively backwards, starting by setting the level of assets in $T + 1$ to zero, and then updating the continuation value at each point in time, according to the optimal decisions between the final period and the current period. While the optimal policies are found by going backwards, the actual solution paths for each individual $i \in \{1, 2, \dots, I\}$ are found by applying the linearly interpolated policy rules going forward from the beginning of life to the end of life. At the beginning of life, individuals differ only in the amount of initial wealth they are endowed with. During lifetime they differ with respect to the their individual histories of idiosyncratic income shocks and business risk realizations. This explains the distribution of solution paths over assets and consumption in the population of I individuals.[4]

Having obtained the solution paths for the entire population according to individual draws of initial wealth and shock histories, it is straightforward to aggregate and to calculate statistics for the population. The basic problem is to translate a set of longitudinal observations for a population of individuals into a representative cross-section at a given point in time. Aggregate uncertainty is handled by randomizing overall possible realization paths. First, we make sure that the age-structure in the observed population (as taken from the CPS) corresponds to the age-structure of the simulated economy. Therefore, starting from the longitudinal simulated paths for the I individuals, we split the sample into T age groups, such that for each age its share in the population corresponds to its share in the CPS. Second, the simulated values for each age group are translated into a common unit of account, by adjusting for the aggregate real income growth rate. Aggregation and the number of individuals do not increase computation time much, once the optimal decision rules are computed, the marginal computational cost of further numerical analysis is low.

The application of dynamic programming techniques in this case requires a special structure, due to the informational assumptions specific to returns on private equity. In the computation of optimal decisions we must distinguish two groups of agents. The first group are the entrepreneurs, who can observe business risk, for whom we compute optimal decisions conditional on the observed realizations of uncertainty in both the dimension of business risk and idiosyncratic income risk. The second group is that of the non-entrepreneurs, defined as holding zero entrepreneurial assets, for whom policy rules are computed conditionally on the observed realizations of idiosyncratic income shocks only, while using the unconditional distribution over business risk. When simulating the optimal paths for the individuals those two sets of rules must be applied appropriately, depending on whether at the point of decision

[4] We simulate a population of $I = 100{,}000$ individuals to compute our statistics.

the individual does hold the entrepreneurial asset or not. The kink in the value function due to the existence of startup costs is handled by computing two separate value functions for each node on the grid at which starting a business is feasible and using the envelope of the two functions as the continuation value.

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