



Bank capital regulation with random audits

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Abstract

We consider a model of optimal bank closure rules (cum capital replenishment by banks), with Poisson-distributed audits of the bank's asset value by the regulator, with the goal of eliminating (ameliorating) the incentives of levered bank shareholders/managers to take excessive risks in their choice of underlying assets. The roles of (tax or other) subsidies on deposit interest payments by the bank, and of the auditing frequency are examined. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

The topic of prudential regulation of banking has received much attention in both theoretical and policy domains over the last decade or so. On the policy side, we have progressed from the rapid deregulation of controls on banks' asset portfolios and deposit interest rates in the early 1980s, to re-thinking about sensible regulations especially on bank capital requirements as well as on monitoring and closure rules in the 1990s, epitomized by the Basle accords on capital ratios recently extended to 'market risks' on traded

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asset portfolios. On the theoretical side, interest in these issues has been rekindled by recent advances in the microeconomic modeling of banks (see Bhattacharya and Thakor, 1993, for a survey), and also on capital structure and optimal (contingent) control rules for corporate governance of a levered firm; see Fischer et al. (1989), Hart (1996), and Dewatripont and Tirole (1994) for applications to bank regulation.

The policy debate on bank regulation—whereby regulatory authorities serve as a “proxy” for both (a) dispersed bank deposit holders subject to free-rider problems in monitoring their banks’ asset choices and returns performances, and (b) the general public concerned with the “contagion effects” of a bank’s failure on other related banks and the payment system—has occurred with the backdrop of heightened instabilities in the banking systems of many countries (the US in the 1980s, and Japan today). The market environment facing banks has also changed quite dramatically since the 1970s, beginning with disintermediation arising from the advent of money market funds for short-maturity deposits, to increased interbank competition in the deregulated environment of the 1980s, through the explosive growth of derivative asset markets that allow banks much greater flexibility in hedging their asset portfolios risks, and also to speculate on such risks in the economy as whole.

A few key insights have emerged in the realm of bank regulation over this period, and some of these have been reflected in policies or in proposals for regulatory reform. First, the role of equilibrium rents in the banking market, both in protecting banks from default given risks and, more importantly, in creating incentives for bank managers to wisely choose their asset portfolio risk, has acquired credence since the empirical findings of Keeley (1990). Second, perhaps most importantly in the policy domain, the role of bank capital (regulations) in reducing the default risks on deposits, as well as in incentivising sensible risk-taking, has acquired widespread recognition, as a minimally intrusive and verifiable instrument of prudential regulation. Third, other controls and regulations on interbank competition—such as bank charter/entry rules, ceilings on bank deposit interest rates, and even portfolio restrictions have begun to be closely examined as potentially welfare-enhancing components of regulatory policy. Fourth, various aspects of the regulators’ functioning itself, such as their inability to precommit to tough closure rules given their instinct for self-preservation of reputations by not taking strong actions early, have also received significant research attention; see Sabani (1996).

However, while some progress has occurred along qualitative lines, ‘multi-instrument and rich’ models of prudential regulation which incorporate detailed quantitative criteria for the use of different instruments of regulation, have not been advanced in great numbers. In the domain of bank capital regulation, Hellwig (1998) has observed that the Basle Accord capital ratio regulations take no account of dynamic risks, and hence the (regulated) readjustment of equity capital in a multi-period context. He has also noted that

for the ‘non-market’ risks of not-easily-liquidated assets the precise levels of capital regulations are not carefully justified; ‘why 8%, rather than 4% or indeed 50%?’, he asks.

Some research over the past two decades has sought to address these structural and quantified issues pertinent to prudential bank regulation and the several alternative instruments thereof. In Bhattacharya (1982), it was pointed out that future rents in banking, arising from margins between risk-adjusted lending and deposit interest rates, would help curtail bank equityholders’ (managers’) incentives to take excessive risks in the short-run, and that the requisite level of rents for efficient risk-taking need not be preserved in a competitive environment, in which raising its deposit interest rate would cause a higher volume of funds to be available to a given bank. More recently, Hellmann et al. (1997) have extended this line of enquiry to incorporate bank equity capital ratio regulations as well. Employing the strong assumption that the (risk-adjusted) cost of bank equity capital exceeds the return on any investment opportunities available to the bank, they reach the conclusion that regulating banks through capital controls alone is never optimal, and controls (ceilings) on the levels of interest rates paid on bank deposits are called for to encourage efficient risk-taking.

The papers of Gehrig (1997) and Matutes and Vives (1997) also model interbank competition and its impact on banks’ default probabilities, with a focus on regulatory restriction to improve on the welfare impact of the market outcomes. Gehrig (1997) models imperfect Hotelling-type locationally differentiated competition across banks in both their asset/loan and deposit markets, with entry and rents determining bank survival probabilities given both idiosyncratic and systemic default risks on loans. However, there is no further margin of choice among differentially risky assets/loans by the banks in his model. Given this, regulatory instruments such as deposit interest rate controls have no role in terms of enhancing future rents to induce prudent current risk-taking by banks—instead these may be used to induce greater entry by new banks when this is too low in market equilibrium. In the paper of Matutes and Vives (1997), only the deposit-market competition is explicitly modeled, but banks have choices among alternative mean-preserving asset return distributions. Default by a bank results in exogenously given social costs which their model does not endogenize. Regulatory responses may have a role in reducing such costs, given the level of default risks of banks in market equilibrium. However, instruments such as deposit interest rate controls only affect default risks directly by creating profit buffers for banks, which are nevertheless induced to choose their most risky investment strategies.

Some other papers have taken a more explicitly quantitative ‘valuation approach’ to the question of prudential regulation of banks. In two early papers, Merton (1977, 1978) established the isomorphism between the value of a deposit insurer’s liabilities and that of a put option on the bank’s

assets, first without and then with random and costly interim monitoring of the bank's asset value. However, these models omitted the possibility of ongoing equity capital replenishment as well as regulatory choice of the level of the closure rule, the (interim) asset value at which the bank would be closed. Issues concerning endogenous choices of their asset portfolio risks by banks were also not fully addressed. These issues of regulatory choice of a bank closure rule, and its impact on the choice of risk-taking by the bank's equityholders/managers, have received more attention in the recent research of Fries et al. (1997). However, their bank closure rule is predicated on a bankruptcy/reorganization cost criterion which has the feature that, given the scale of the deposit liabilities, the regulator's bankruptcy/closure costs are lower if the bank's asset value is lower at the time of its closure! Hence, the regulator "waits for the bank to shrink away", subject to a given monitoring cost rate while doing so. With a more sensible bank closure/reorganization cost function whereby the regulator is better off having a higher bank asset value base at the time of closure, and continuous monitoring of the continuously-evolving bank asset value, issues of default risk on bank deposits would simply not arise.

In this paper, we seek to address and quantify the issues that have been raised in prior literature in a systematic way, to incorporate (a) ongoing deposit liabilities (perpetual debt) and the possibility of a bank's reorganization following default by one set of equityholders; (b) the possibility of replenishment of bank capital—although not as instantaneously as in Hellmann et al. (1997)—by equityholders subject to the constraint of a closure rule imposed by the regulator; (c) subsidies (from tax-shield or controlled interest rates, for example) on deposit finance relative to the costs of bank equity capital; and (d) stochastic Poisson audits of variable intensity by a bank regulator when the underlying asset value of a bank follows a continuous-time diffusion process. The regulatory controls are chosen to eliminate any excessive risk-taking incentives of the levered bank equityholders, at least in the region where the bank would not be closed on audit. Our model enables us to quantify the tradeoffs among various policy instruments in attaining this goal. We hope that this analysis will stimulate further research on richer models that incorporate considerations of the relative costs and benefits of the alternative policy instruments such as bank closure rules, the levels of subsidies/ceilings on bank deposit interest rates, the levels of capital requirements and the speed of replenishment thereof, and regulatory audit frequency. A more complete analysis should take into account the implications of various bank regulations for expected reorganization costs, and the effects on the volume of savings and its channeling via bank-intermediated financing.

Our paper is organized as follows. In the next section, we describe the structure of our model of the valuation of the liabilities of a bank subject to regulatory controls on closure, and derive analytical (though highly non-linear

in some parameters) solutions for these as well as for the optimal regulatory closure rule, which seeks to induce neutrality towards risk-taking by bank equityholders. In Section 3, we calibrate the model numerically, for a range of values of endogenizable variables such as the maximal payout ratio (which determines the speed of bank equity replenishment as well), the regulatory audit frequency, the differentially lower cost of deposit finance, and the (overall value-maximizing) choice of the level of riskiness of the returns on the bank's asset portfolio. In Section 4, we conclude, with suggestions for further research on the regulatory issues that we attempt to highlight.

2. The model

As in Merton (1974), Black and Cox (1976), and Leland (1994), we assume that the bank's unlevered asset value V follows a continuous-time diffusion process characterized by the following stochastic differential equation:

$$\frac{dV}{V} = [\mu(V, t) - \delta] dt + \sigma dz, \quad (1)$$

where $\mu(V, t)$ is the total expected rate of return on asset value V , δ is the constant fraction of the asset value paid out to security holders, σ is the constant proportional volatility of the asset value per unit time, and dz is the increment of a standard Brownian motion. This stochastic process continues without any time limit unless the asset value V falls below a default-triggering value B^* chosen by the bank's equityholders, or there occurs an audit and the regulatory authority decides to close the bank because V is below a pre-specified value B which is set by the regulatory body. The regulator's audits of the bank's asset value are stochastic and follow a Poisson process where the mean number of audits per unit time is denoted by λ . Formally, this is described by the following stochastic process

$$dA = dq, \quad (2)$$

where A is one if an audit occurs and zero otherwise. Note that under the above assumptions the probability that an audit occurs in the time interval dt is λdt , the probability of no audit is $1 - \lambda dt$ and the probability of more than one audit is of order $O(dt)$. It is assumed that the two stochastic processes dz and dq are independent. Later on it will be shown that the two closure rule parameters B^* and B can be determined analytically as a part of the regulatory framework.

2.1. Value of debt

In order to price the debt of the bank, let us assume that a riskless asset exists that pays a constant rate of interest r and that the bank continuously

pays a non-negative coupon C per unit of time to its creditors (depositors), unless it declares default or is closed by the regulatory authority. The value of the outstanding debt $D(V, A; B^*, B, \sigma, r, C, \lambda, \delta)$ is given by the solution of the following non-linear ordinary differential equation

$$\frac{1}{2}\sigma^2 V^2 D_{VV} + (r - \delta)VD_V - rD + C + \mathbf{1}_{[B^*, B]}\lambda(V - D) = 0, \tag{3}$$

which must hold if either (a) all agents are risk-neutral, or (b) agents other than the regulator are always perfectly informed of these asset and liability values and can trade continuously in these. Furthermore, the following economic boundary conditions must also be satisfied:

- (i) $\lim_{V \rightarrow \infty} D(V) = C/r,$
- (ii) $\lim_{V \rightarrow B^+} D(V) = \lim_{V \rightarrow B^-} D(V),$
- (iii) $\lim_{V \rightarrow B^+} D_V(V) = \lim_{V \rightarrow B^-} D_V(V),$
- (iv) $D(B^*) = B^*.$

Condition (i) holds because default becomes irrelevant as V becomes large and the value of debt approaches the value of the capitalized coupon and therefore the value of risk-free debt. Conditions (ii) and (iii) are the common ‘smooth pasting’ conditions, and condition (iv)—as well as the last term in Eq. (3)—guarantee that in the case of default—or closure by the regulator—the value of debt is equal the asset value of the bank. The solution of the above differential equation is given by

$$D_1(V) = \frac{C}{r} + \alpha_1 V^{x_1} + \alpha_2 V^{x_2} \tag{4}$$

as long as $V \geq B$ and it is

$$D_2(V) = \frac{C}{r + \lambda} + \frac{\lambda}{\lambda + \delta}V + \beta_1 V^{y_1} + \beta_2 V^{y_2} \tag{5}$$

for $B^* \leq V \leq B$.

Simple algebraic calculations show that

$$x_{1,2} = \frac{\sigma^2 - 2(r - \delta) \pm \sqrt{[\sigma^2 - 2(r - \delta)]^2 + 8r\sigma^2}}{2\sigma^2}, \tag{6}$$

$$y_{1,2} = \frac{\sigma^2 - 2(r - \delta) \pm \sqrt{[\sigma^2 - 2(r - \delta)]^2 + 8(r + \lambda)\sigma^2}}{2\sigma^2} \tag{7}$$

and without loss of generality we can assume that x_1 and y_1 are negative. It is immediately clear that condition (i) implies that $\alpha_2 = 0$, and the remaining conditions yield the following expressions for α_1, β_1 and β_2 :

$$\beta_1 = \frac{B^{*y_2}[\lambda C x_1 / r(\lambda + r) + (\lambda(1 - x_1)B / (\lambda + \delta))] - (x_1 - y_2)B^{y_2}[\delta B^* / (\lambda + \delta) - (C / (\lambda + r))]}{(x_1 - y_1)B^{y_1}B^{*y_2} - (x_1 - y_2)B^{*y_1}B^{y_2}}, \tag{8}$$

$$\beta_2 = - \frac{B^{*y_1} [\lambda C x_1 / r (\lambda + r) + (\lambda (1 - x_1) B / (\lambda + \delta))] - (x_1 - y_1) B^{y_1} [\delta B^* / (\lambda + \delta) - (C / (\lambda + r))]}{(x_1 - y_1) B^{y_1} B^{*y_2} - (x_1 - y_2) B^{*y_1} B^{y_2}}, \tag{9}$$

$$\alpha_1 = \frac{\lambda B^{1-x_1}}{x_1 (\lambda + \delta)} + \frac{\beta_1 y_1 B^{y_1-x_1}}{x_1} + \frac{\beta_2 y_2 B^{y_2-x_1}}{x_1}. \tag{10}$$

So far, B^* and B are exogenously given parameters. In what follows, we shall show how these can be endogenized.

2.2. Value of subsidy

Our model allows for the fact that the banking industry may not be perfectly competitive either because of collusion, interest ceilings on deposits, or government (tax-shield on interest) subsidies. Specifically, we assume that the size of this rent is proportional to the interest paid to depositors, τC . This implies that the value of the subsidy, S , obeys the following non-linear ordinary differential equation:

$$\frac{1}{2} \sigma^2 V^2 S_{VV} + (r - \delta) V S_V - r S + \tau C - \mathbf{1}_{[B^*, B]} \lambda S = 0, \tag{11}$$

where the following economic boundary conditions must be satisfied:

- (i) $\lim_{V \rightarrow \infty} S(V) = \tau C / r$,
- (ii) $\lim_{V \rightarrow B^+} S(V) = \lim_{V \rightarrow B^-} S(V)$,
- (iii) $\lim_{V \rightarrow B^+} S_V(V) = \lim_{V \rightarrow B^-} S_V(V)$,
- (iv) $S(B^*) = 0$.

Condition (i) holds because default becomes irrelevant as V becomes large and the value of the subsidy approaches its riskless capitalized present value. Conditions (ii) and (iii) are the common smooth pasting conditions, and condition (iv) reflects the loss of the (tax-) subsidy benefits as far as the current owners of the bank are concerned if the bank declares default or if it is closed. The solution of the above differential equation is given by

$$S_1(V) = \frac{\tau C}{r} + \tilde{\alpha}_1 V^{x_1} + \tilde{\alpha}_2 V^{x_2} \tag{12}$$

as long as $V \geq B$ and it is

$$S_2(V) = \frac{\tau C}{r + \lambda} + \tilde{\beta}_1 V^{y_1} + \tilde{\beta}_2 V^{y_2} \tag{13}$$

for $B^* \leq V \leq B$, where $x_{1,2}$ and $y_{1,2}$ are given as before.

The boundary condition (i) implies that $\tilde{\alpha}_2 = 0$ and the remaining ones yield the following three equations:

$$\tilde{\beta}_1 = \frac{\tau C [(\lambda x_1 B^{*y_2}) / r + (x_1 - y_2) B^{y_2}]}{(\lambda + r) [(x_1 - y_1) B^{y_1} B^{*y_2} - (x_1 - y_2) B^{*y_1} B^{y_2}]}, \tag{14}$$

$$\tilde{\beta}_2 = - \frac{\tau C[(\lambda x_1 B^{*y_1})/r + (x_1 - y_1)B^{y_1}]}{(\lambda + r)[(x_1 - y_1)B^{y_1}B^{*y_2} - (x_1 - y_2)B^{*y_1}B^{y_2}]}, \tag{15}$$

$$\tilde{\alpha}_1 = \frac{\tilde{\beta}_1 y_1 B^{y_1 - x_1}}{x_1} + \frac{\tilde{\beta}_2 y_2 B^{y_2 - x_1}}{x_1}. \tag{16}$$

As before, the parameters B^* and B are exogenous but they will be determined endogenously in what follows. We are now in a position to define the total value, TV , of the bank.

2.3. Total value of the bank and equity value

The total value of the bank $TV(V)$ is the bank’s asset value plus the value of the subsidy of coupon payments:

$$TV(V) = V + S(V). \tag{17}$$

The value of equity is the total value of the bank minus the value of its debt:

$$E(V) = V + S(V) - D(V). \tag{18}$$

More explicitly, we have

$$E_1(V) = \frac{C(\tau - 1)}{r} + V + (\tilde{\alpha}_1 - \alpha_1)V^{x_1} \tag{19}$$

for $V \geq B$ and

$$E_2(V) = \frac{C(\tau - 1)}{r} + \frac{\delta V}{\lambda + \delta} + (\tilde{\beta}_1 - \beta_1)V^{y_1} + (\tilde{\beta}_2 - \beta_2)V^{y_2} \tag{20}$$

for $B^* \leq V \leq B$.

Since the bankruptcy-triggering asset value level B^* is chosen by the bank equityholders (rather than imposed by a covenant), then as pointed out by Merton (1973) this value is determined by the following ‘low contact’ condition:

$$E_V(B^*) = 0. \tag{21}$$

This condition yields the following equation:

$$\frac{\delta B^*}{\lambda + \delta} + (\tilde{\beta}_1 - \beta_1)y_1 B^{*y_1} + (\tilde{\beta}_2 - \beta_2)y_2 B^{*y_2} = 0. \tag{22}$$

The regulatory closure rule B , whereby the bank is closed and reorganized if a situation of $V < B$ is discovered during a (Poisson-distributed) regulatory audit, implicitly defines the minimum capital adequacy standard. For empirical plausibility, the modeled regulatory regime requires that the total value of the bank assets relative to the face value of its liabilities when $V = B$ should not exceed unity. We require the regulatory authority to choose the closure rule B

such that the bank's equityholders become indifferent with respect to the risk σ the bank takes, for all asset values $V \geq B$. Mathematically, this implies:

$$\frac{\partial E_1(V)}{\partial \sigma^2} \equiv 0. \quad (23)$$

Generally, the critical B which satisfies this condition will depend on the level of σ .¹ One interpretation is that the regulator announces the function $B(\sigma)$, and then the bank picks σ . Thus, the practical closure rule implied by the model accords quite well with the regulatory rules imposed by many OECD countries, for example, by the EU capital adequacy directives. These specify that the bank's equity requirements are directly proportional to its value at risk which in our framework is proportional to σ .²

By concentrating on the requirement that risk shifting incentives are completely eliminated by the closure rules, this paper models a stylized objective function of the regulator. In a richer setting than the one modeled here, regulators should take into account their monitoring costs, and the deadweight costs of deposit insurance and bank reorganization. In a more general model, changes of the riskiness of a bank's operations would also influence other parameters, such as its asset value. If this effect is very strong, then eliminating risk-shifting incentives to deviate from the overall value-maximizing investment choice would be most desirable. In general, the implementation of any regulatory strategy will require that it is incentive compatible for the banks' equityholders not to alter the investment strategies on which the regulations are predicated.

The condition that the first derivative of the equity value with respect to a change in the volatility be zero holds if and only if

$$\tilde{\alpha}_1 = \alpha_1, \quad (24)$$

which is the last equation we need to determine the model parameters B and B^* endogenously. Using Eqs. (8) and (14) above, the above condition (24)

¹ One way to justify the regulatory closure rule adopted in the following analysis is to assume that the bank's risk level is observable but not verifiable in a court ex post. The regulator can choose B based on his assessment of bank risk ex post. More extreme sanctions for excessive risk taking, such as jail terms are unlikely to be feasible. Complications would be introduced when the riskiness of a bank's asset/loan portfolio could deteriorate owing to luck, rather than via explicit asset substitution by its managers or equity holders. If the bank's regulator cannot distinguish that from unlucky shifts in σ then the optimal regulatory $B(\sigma)$ function of our model should always be implemented, if the potential societal losses from σ -choices which would not maximize V are sufficiently large, relative to any costs arising from the bank managers' risk-aversion, or the (unmodeled) costs of bank closure or reorganization.

² A bank's capital requirements on a given day are actually calculated on the basis of the maximum between the current value at risk and a historic average value at risk, e.g., over the past 60 days.

can be rewritten as follows:

$$(\tilde{\beta}_1 - \beta_1)y_1B^{y_1} + (\tilde{\beta}_2 - \beta_2)y_2B^{y_2} - \frac{\lambda B}{\lambda + \delta} = 0. \tag{25}$$

After some tedious algebraic manipulations, we can show that B and B^* are the solutions to the following (highly) non-linear two-dimensional system of equations:

$$[\tilde{\beta}_1(B, B^*) - \beta_1(B, B^*)]y_1B^{y_1} + [\tilde{\beta}_2(B, B^*) - \beta_2(B, B^*)]y_2B^{y_2} - \frac{\lambda B}{\lambda + \delta} = 0, \tag{26}$$

$$[\tilde{\beta}_1(B, B^*) - \beta_1(B, B^*)]y_1B^{*y_1} + [\tilde{\beta}_2(B, B^*) - \beta_2(B, B^*)]y_2B^{*y_2} + \frac{\delta B^*}{\lambda + \delta} = 0. \tag{27}$$

It is not clear whether or not a solution exists, and if so if it has the economic property that $B \geq B^* \geq 0$. The existence of such a solution has to be proved, of course. In order to solve the above system of equations, it turns out that a substitution of the form $u = B/B^*$ is useful. With this new variable the above equations have the following functional form:

$$\frac{B^*[\delta x_1(y_1 - y_2)u^{y_1+y_2} - \lambda x_1u[(1 - y_1)u^{y_1} - (1 - y_2)u^{y_2}]]}{\lambda + \delta} + \frac{C(\tau - 1)[x_1(y_1 - y_2)u^{y_1+y_2} + (\lambda/r)x_1(y_1u^{y_1} - y_2u^{y_2})]}{\lambda + r} = 0, \tag{28}$$

$$\frac{B^*[\delta[(x_1 - y_1)(1 - y_2)u^{y_1} - (x_1 - y_2)(1 - y_1)u^{y_2}] - \lambda u(1 - x_1)(y_1 - y_2)]}{\lambda + \delta} + \frac{C(\tau - 1)[y_1(x_1 - y_2)u^{y_2} - y_2(x_1 - y_1)u^{y_1} + (\lambda/r)x_1(y_1 - y_2)]}{\lambda + r} = 0. \tag{29}$$

These equations are linear in B^* and non-linear in u . Therefore, we can write down B^* as a function of u and decouple u from B^* . Doing this we get

$$B^* = \frac{C(1 - \tau)(\lambda + \delta)}{\lambda + r} \times \frac{[x_1(y_1 - y_2)u^{y_1+y_2} + (\lambda/r)x_1(y_1u^{y_1} - y_2u^{y_2})]}{[\delta x_1(y_1 - y_2)u^{y_1+y_2} - \lambda x_1u[(1 - y_1)u^{y_1} - (1 - y_2)u^{y_2}]]}. \tag{30}$$

Thus, we are left with only one non-linear equation in u :

$$\frac{y_1(x_1 - y_2)u^{y_2} - y_2(x_1 - y_1)u^{y_1} + (\lambda/r)x_1(y_1 - y_2)}{\delta[(x_1 - y_1)(1 - y_2)u^{y_1} - (x_1 - y_2)(1 - y_1)u^{y_2}] - \lambda u(1 - x_1)(y_1 - y_2)} - \frac{x_1(y_1 - y_2)u^{y_1+y_2} + (\lambda/r)x_1(y_1u^{y_1} - y_2u^{y_2})}{\delta x_1(y_1 - y_2)u^{y_1+y_2} - \lambda x_1 u[(1 - y_1)u^{y_1} - (1 - y_2)u^{y_2}]} = 0. \quad (31)$$

It is easy to show that the left-hand side of (30) is always positive and hence that B^* is always positive. Eq. (31) is independent of C and τ and, therefore, also u does not depend on these parameters. These observations yield immediately the following result.

- Theorem 1.* (i) *There exists a solution (B, B^*) with the property that $B > B^* > 0$ iff Eq. (31) has a solution $u > 1$.*
 (ii) *B and B^* are linear in the coupon C .*
 (iii) *B and B^* are linear in the subsidy rate τ .*
 (iv) *B and B^* do not depend on V .*

We prove the existence of a solution $u > 1$ for the case $\delta = 0$. (The case $\delta \neq 0$ is similar but the necessary calculations are rather cumbersome.)

Proposition 1. *Assume that there are no net cash outflows, i.e., $\delta = 0$. Then the non-linear equation (31) has a solution $u > 1$.*

Proof. For $\delta = 0$, Eq. (31) attains the following form

$$\begin{aligned} f(u) &\equiv \frac{y_1(x_1 - y_2)(1 - y_2)u^{2y_2}}{(1 - x_1)(y_1 - y_2)} + \frac{y_2(x_1 - y_1)(1 - y_1)u^{2y_1}}{(1 - x_1)(y_1 - y_2)} \\ &\quad - \frac{[y_1(x_1 - y_2)(1 - y_1) + y_2(x_1 - y_1)(1 - y_2) - (y_1 - y_2)^2(1 - x_1)]u^{y_1+y_2}}{(1 - x_1)(y_1 - y_2)} \\ &\quad - \frac{\lambda(x_1 - y_1)u^{y_1}}{r(1 - x_1)} + \frac{\lambda(x_1 - y_2)u^{y_2}}{r(1 - x_1)} = 0. \end{aligned}$$

A simple calculation shows that

$$f(1) = \frac{(y_2 - y_1)}{1 - x_1} + \frac{\lambda(y_2 - 1)}{r(1 - x_1)} < 0$$

and that $\lim_{u \rightarrow \infty} f(u) = \infty$. These two properties of the function $f(u)$ together with its continuity imply that there exists a $u > 1$ which solves the equation. \square

An immediate consequence of Theorem 1 is that there exists (it is easy to write down an explicit expression for these, but this is not done due to the length of the expressions) parameters $a_1, b_1, \tilde{b}_1, b_2, \tilde{b}_2$ which are independent of the coupon rate C such that the following equations hold:

$$D_1(V) = \frac{C}{r} \left[1 - a_1 \left(\frac{V}{C} \right)^{x_1} \right], \quad (32)$$

$$D_2(V) = \frac{\lambda V}{\lambda + \delta} + \frac{C}{r} \left[\frac{r}{\lambda + r} - b_1 \left(\frac{V}{C} \right)^{y_1} - b_2 \left(\frac{V}{C} \right)^{y_2} \right], \quad (33)$$

$$TV_1(V) = V + \frac{C}{r} \left[\tau - a_1 \left(\frac{V}{C} \right)^{x_1} \right], \quad (34)$$

$$TV_2(V) = V + \frac{C}{r} \left[\frac{\tau r}{\lambda + r} - \tilde{b}_1 \left(\frac{V}{C} \right)^{y_1} - \tilde{b}_2 \left(\frac{V}{C} \right)^{y_2} \right], \quad (35)$$

$$E_1(V) = V - \frac{C(1 - \tau)}{r}, \quad (36)$$

$$E_2(V) = \frac{\delta}{\delta + \tau} V - \frac{C}{r} \left[(1 - \tau) - (\tilde{b}_1 - b_1) \left(\frac{V}{C} \right)^{y_1} - (\tilde{b}_2 - b_2) \left(\frac{V}{C} \right)^{y_2} \right], \quad (37)$$

where the indices 1 and 2 indicate the appropriate domain $[B^*, B]$ and $[B, \infty)$, respectively.

3. Numerical results

We will now present numerical comparative static results. For this purpose, we have calibrated the parameters of the model to represent realistic values.³ Unless otherwise stated, all comparative static results in this section are based

³ To obtain a realistic estimate for the volatility of the asset value, we have used COMPUTSTAT information on commercial banks which had a full record of data for the time period 1989–1998. We have then estimated the annual standard deviation of the total asset value. The average value was slightly over 14%. Our base case uses a value of 10%. We have also calculated the average ratio of dividend to total asset value for COMPUSTAT banks for the period 1989–1998. We found that this ratio was 0.37%. However, in our model δV represents the dividend payout plus the interest paid to debtholders. Based on a riskless interest rate of 5% and reasonable leverage ratios (between 80% and 90%), the value used for δ of 4.2%, as assumed in our base case, is realistic. Finally, we assume that the interest rate offered to bank deposits is 10% lower than the equivalent money market interest rate, i.e., $\tau = 10\%$.

on the following parameter values:

Parameter	Value
C	70
r	5%
δ	4.2%
λ	3
τ	10%
σ	10%

In this base case scenario, the par value of the deposits C/r amounts to 1400, and the asset value V at which the net dividend, $\delta V - (1 - \tau)C$, becomes negative is 1500. The asset value V at which the equityholders would stop the interest payments and declare default, B^* , equals 1221.96, and the asset value at which the regulator should choose to close the bank, B , is equal to 1308.04. The latter value would have equaled $1400(1 - 0.1) = 1260$ in the model of Leland (1994), in which continuous observability of the asset value V is assumed; with no subsidy on debt interest payments, the optimal closure rule in that model would simply set $B = 1400$, to keep the debt claim riskless. Note that, unlike in Leland's model, our regulator can only ensure risk-invariance of the equity value E for $V > B$, provided also that she knows the level of risk that the bank's equityholders would choose (optimally, in order to maximize the overall asset value V). The reason for this is that the chosen level of risk would impact on the bank's equityholders' choice between continuing to make coupon payments versus declaring default in the region where $V < B$, but this is as yet not detected by the regulator's audits, and this choice would in turn affect their equity value in the range $V > B$. Thus, our criterion for the regulator's choice of a closure rule presumes that (a) the asset risk level is chosen irrevocably when the bank is still 'solvent' ($V > B$), (b) the regulator has knowledge of the bank's risk-choice and thus she can adapt her closure rule B to reflect this choice by equityholders, so as to induce the optimal (the overall asset value-maximizing) choice of asset return risk. Such a scenario is relevant when the bank's risk choice is 'observable (by the regulator) but not verifiable', so that stronger contractual remedies would be infeasible. It also presumes a certain degree of regulatory discretion regarding bank closure, exercised in the interest of overall value maximization, which may require that the regulator is separated from an agency that insures the (par) value of the bank debt/deposits.

3.1. Comparative statics of debt value and equity value

As can be seen from Fig. 1, both B (solid line) and B^* (dotted line) are linearly increasing functions of the coupon rate C . The equity value is a

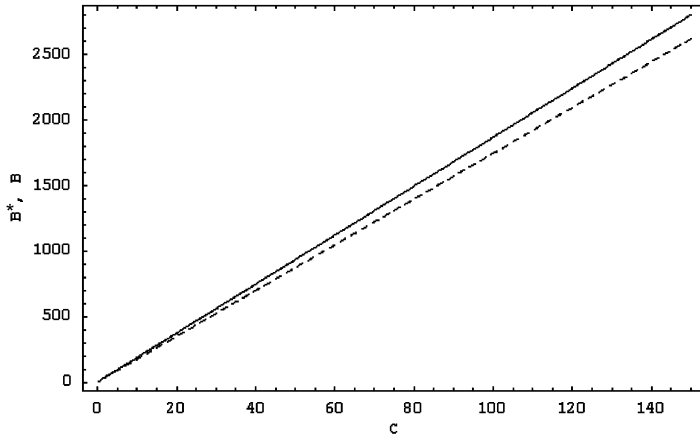


Fig. 1. Closure, default, and the debt coupon.

decreasing function of C , since a higher coupon rate raises the debt (deposit) value, and the increased default cum closure probability associated with a higher coupon reduces the value of the subsidy in proportion to the debt value.

As the (maximum) payout ratio increases, the regulatory closure level B which eliminates risk-shifting incentives first increases and then decreases. The latter effect occurs because the high payout lowers the drift rate on the bank's asset value, so that equityholders voluntarily declare default at higher levels of V . The resulting loss of the value of the subsidy makes a risk increase more costly to equityholders, thereby allowing for a lower regulatory closure level. Both these effects are depicted in Fig. 2 below, in which it is remarkable that the overall impact of the payout ratio on the optimal regulatory closure rule, B , is miniscule (compared to its effect on B^* , for example).

From the regulator's perspective, an important policy instrument aside for capital controls is the audit frequency, or its Poisson intensity. Since auditing, the bank more frequently lowers the risk of large asset and debt value losses, the regulator optimally chooses a lower closure rule B with a higher frequency of audits. The opposite is true for equityholders' choice regarding default. Since a higher audit frequency increases the probability of losing future subsidies if the bank is insolvent, when $B^* < V < B$, the equityholders optimally default at higher asset value levels, B^* s, as the audit frequency increases. Fig. 3 illustrates the numerical magnitudes of these two effects.

The resulting impact of regulatory audit frequency on the bank's debt value is therefore not clear a priori, and this value would impact on the value of the regulator's liabilities if she also insures deposits, as in Merton (1977,

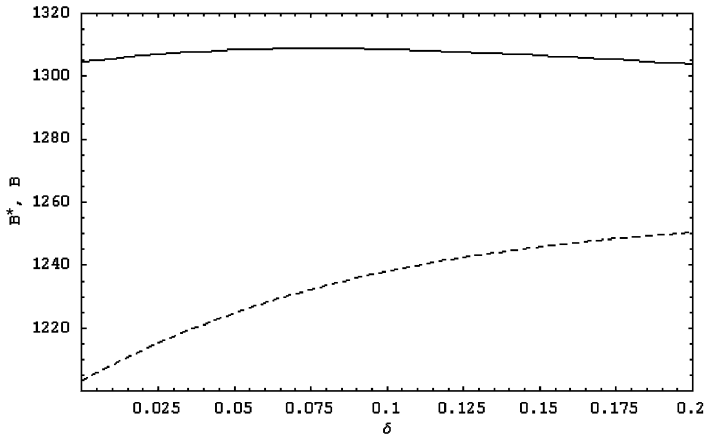


Fig. 2. Closure, default, and the payout ratio.

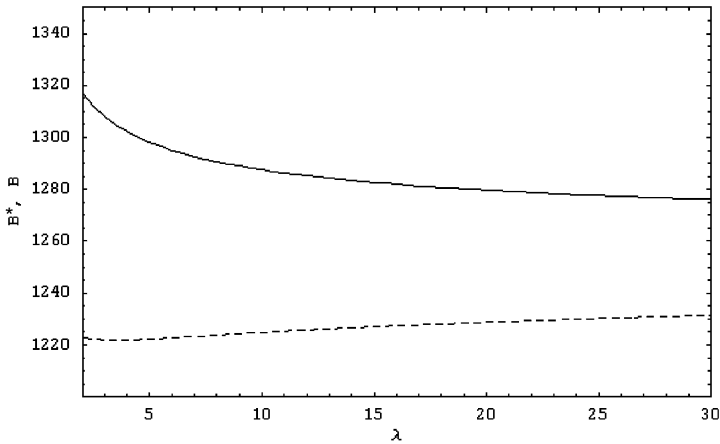


Fig. 3. Closure, default, and the audit frequency.

1978). Our numerical calibrations suggest that the overall impact of higher audit frequency on the debt value is small for a solvent bank, whereas it has a beneficial impact on the debt value in a seriously insolvent bank, as depicted in Figs. 4 and 5 below. The equity value of a solvent bank also shows no significant dependence on audit frequency.

An important question with respect to capital requirements and closure rules is how these should be adjusted to reflect changes in the underlying (and overall value-maximizing) level of riskiness of the bank's asset portfolio. Our numerical results show that the optimal closure level, B , is an increasing

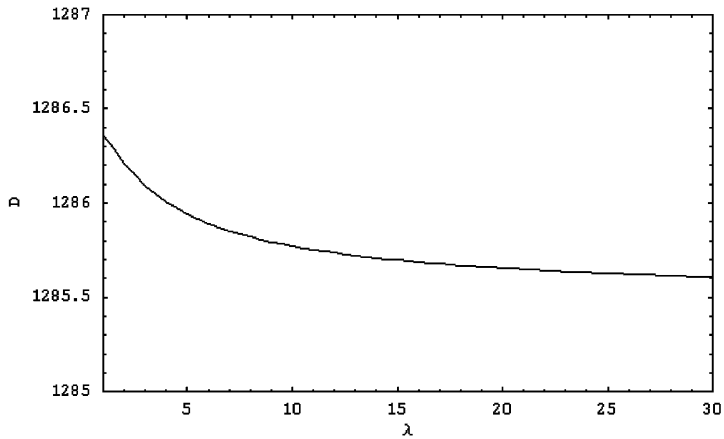


Fig. 4. Debt value and audit frequency (solvent bank).

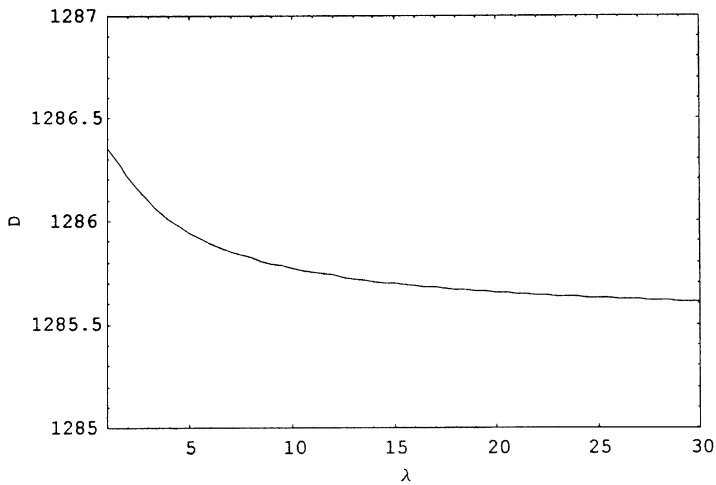


Fig. 5. Debt value and audit frequency (insolvent bank).

function of the asset return volatility, flowing from the related observation that with higher volatility equityholders would voluntarily continue coupon payments at lower asset values, thus lowering B^* . It is also interesting to note, in Fig. 6 below, that the relationship between B and the asset returns volatility is almost linear, as suggested by the value at risk approach for marketable assets, even though the closure criterion in our model is not motivated by fixing the probability of insolvency (over a given horizon) as such.

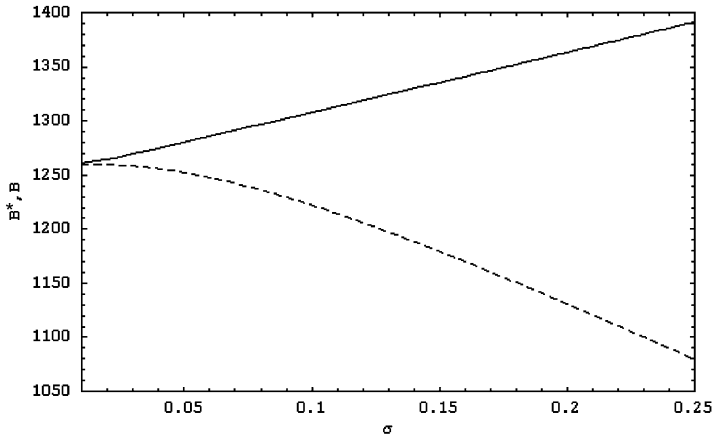


Fig. 6. Closure, default, and asset return volatility.

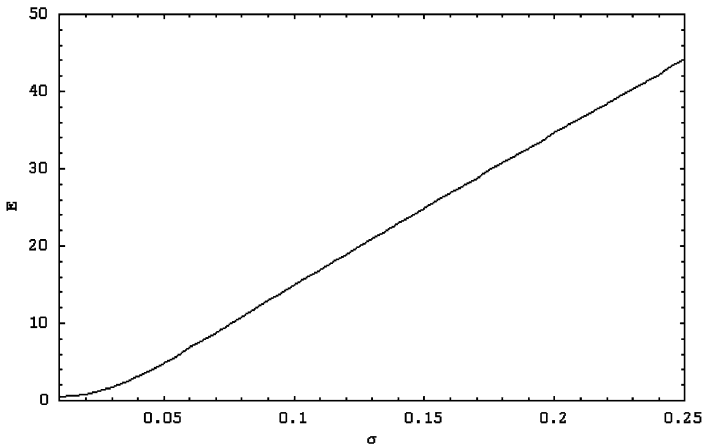


Fig. 7. Equity value and asset return volatility (insolvent bank).

Given the magnitudes of the two effects noted above, increased asset returns volatility still lowers the debt value of a solvent bank. It leaves the equity value of a solvent bank unaffected (by design), as B is adjusted appropriately. Nevertheless, the equity value of an insolvent bank is significantly increasing in its asset return volatility (Fig. 7), so that the regulatory controls depicted here can not eliminate bank equityholders ‘gambling for resurrection’ via asset substitution in the range $B^* < V < B$; only intrusive monitoring of the bank’s activities (when V was close to B in recent audits) might rule this out.

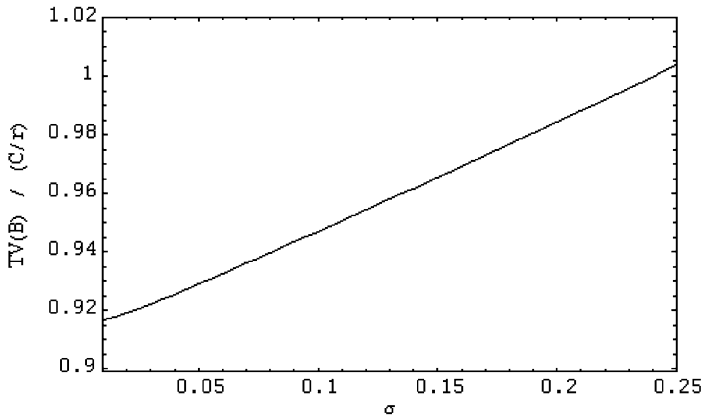


Fig. 8. Total bank value, face value of liabilities, and asset return volatility at closure.

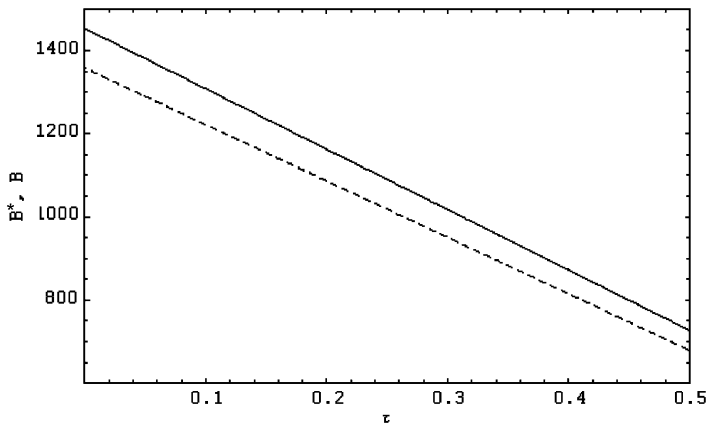


Fig. 9. Closure, default and subsidy.

It is interesting to compare the value of the bank at the closure point $TV(B)$, with the face value of the liabilities, C/r . Numerical simulations show that this ratio is less than one over the entire realistic range of parameter values. Fig. 8 gives the comparative static results for this ratio with respect to changes in the volatility.

Finally, we can explore the relationship between the closure points B and B^* and the subsidy parameter τ . Fig. 9 shows that both the point at which the regulator wishes to close the bank and the point at which the equityholders abandon the bank decrease significantly with τ . Thus, as the bank license becomes more valuable, the equityholders have less incentives to abandon

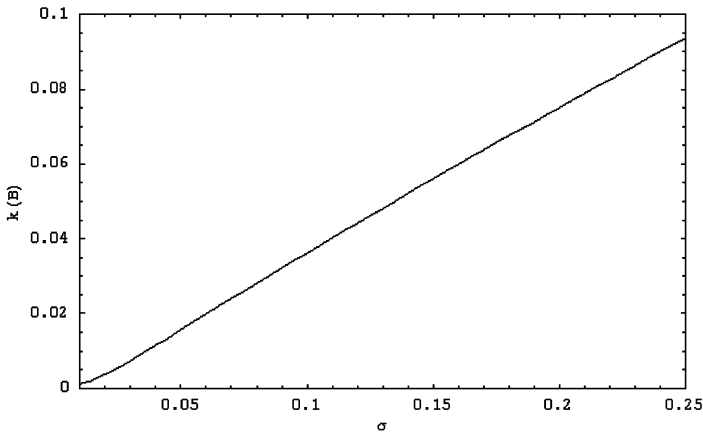


Fig. 10. Market equity ratio at closure, and return volatility.

the firm and a lower closure point suffices to eliminate any risk shifting incentives.

3.2. Bank closure and capital adequacy

The capital adequacy directive regulates the bank's equity requirements in such a way that the bank's qualifying capital, \bar{E} , must exceed a certain percentage of the market value of its assets:

$$\bar{E} \geq kTV(V),$$

where the factor k depends on the risk of the bank's assets. If we use the equity value, $E(V)$, as a proxy for \bar{E} and evaluate the ratio of $E(V)$ to $TV(V)$ at the closure level B , our calibrations suggest that the factor k increases (almost) linearly with the asset volatility σ , which is in accord with the value at risk approach, where $k = \alpha\sigma$ (Fig. 10). For the audit frequency (average of 3 per year) in our base case, k increases from 1.56% to 3.62%, and finally 5.60%, as the annual standard deviation of asset returns grows from 5% to 10% to 15% (the low end of which is more likely), well within the range of the recommended ex ante equity capital ratios in the capital adequacy directive.

However, the measure of capital requirements as defined above does not perfectly conform to the definition stated in the capital adequacy directive. In particular, if the bank's capital falls short of that required level, the bank will not be closed immediately but instead be urged to increase its capital. Therefore, we alternatively calculate the ratio of $E(V)$ to $TV(V)$ at the asset value level $\bar{X} = (1 - \tau)C/\delta$, at which the equityholders are forced to provide

additional capital. This alternative definition of capital requirements yields equity ratios that are considerably above the values defined by the capital adequacy directive. For a payout ratio of 4.2%, which is not very low compared to the riskless required rate of return (on equity) of 5%, this equity ratio exceeds 14% and increases further as the dividend yield, δ , decreases.

4. Conclusion

This paper makes two main contributions to the literature on intertemporal bank regulation. First, we simultaneously consider several regulatory policy instruments. More specifically, the regulatory framework is specified by (i) a capital replenishment rule or an equity value at which equityholders are required to contribute more capital; (ii) a closure rule or an equity value below which the bank is closed if audited; and (iii) a frequency at which a bank is audited. Second, the analysis demonstrates how capital adequacy, bank auditing and closure regulation can be designed to eliminate or mitigate bank equityholders' incentives to take excessive risks in their choice of bank assets. In contrast to existing literature, we demonstrate that a given combination of capital replenishment, closure and auditing regulation completely eliminates risk-taking incentives as long as the bank is solvent.

Numerical examples reveal how the model parameters influence the regulatory policy which leads to optimal incentive compatible risk choices. First, we find that closure levels are increasing almost linearly with the risk of the underlying assets. This provides a rationale that capital regulation should be linked to the bank's market risk, possibly quantified by internal value-at-risk models. Second, somewhat surprisingly, there is no clear cut relationship between capital replenishment rules implied by dividend constraints and optimal closure.

The third policy variable, namely the frequency of bank audits, significantly affects the optimal closure level. Higher frequency of bank auditing allows the regulator to close the bank at lower asset values, without creating adverse risk-shifting incentives. Our analysis also yields numerical values for capital adequacy regulation. We provide solutions for equity market values below which capital replenishment is required, as well as equity market values below which closure takes place upon an audit. For reasonable parameter values, our model produces capital adequacy levels which are significantly higher than the eight percent currently required by the BIS standards. This framework may be extended in several ways. First, the amount of deposits could be made stochastic. This would introduce another source of risk which must be taken into account both by equityholders and regulators. Second, λ could be allowed to change after an audit. In particular, the regulator may want to increase λ after an audit reveals that the bank is close to bankruptcy. Third,

an objective function for the regulator may be specified so that an optimal mix of regulatory instruments can be determined. This would require making assumptions about costs of audits, bankruptcy, etc.

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